International Journal of Human Resource Management Research
Vol. 1, No. 2, December 2013, PP: 10 -51
Available online at http://acascipub.com/Journals.php

Research article

STUDIES ON TWO GRADED MAN POWER MODEL WITH BULK RECRUITEMENT IN BOTH GRADES

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Abstract

Manpower planning is very useful for human resource management in large organizations. In this paper, we develop and analyze a two grade manpower model with direct bulk recruitment in both the grades. Here it is assumed that the organization is having two grades and the recruitment is done in both the grades in bulk. That is, a random number of employees at a time are recruited for both the grades with different recruitment rates. The employee once recruited in first grade, may be promoted to a second grade after spending a random period of time in grade 2 or he may leave the organization with certain probability. The employee after spending a random period of time in grade 2 leaves the organization. The recruitment process of batches follows a Poisson process and hence the recruitment of an employee into the organization is characterized by a compound Poisson process. It is also assumed that the promotion and leaving processes follow Poisson processes. Using the difference differential equations, the joint Probability generating function of the number of employees in each grade under transient conditions is derived. The characteristics of the model are derived under transient and steady state conditions. The performance evaluation of the model is carried through numerical illustration. The sensitivity of the model with respect to the parameters is also studied. This model is more useful in analyzing the manpower situations and is more close to the practical situations arising at government, public and private sector organizations.

Key Words: Graded manpower system; Compound Poisson Process; Joint probability generating function; Bulk recruitment; stochastic process.

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1. Introduction

Manpower models are more useful for policy analysis of human resources in any organization. Manpower planning is useful in business and industrial sectors. Since the human behavior is random in nature, stochastic model provide the basic frame work for efficient analysis and design of a manpower systems. Starting from seal [12] a wide spectrum of manpower planning models have been developed and analyzed with various assumptions in order to analyze the practical situations. Bartholomew [3,4], who is the pioneer in modern manpower planning models suggested and analyzed some manpower models with complete length of service distributions for different graded manpower systems. Silock [11] studied the phenomenon of labour turnover as analogy to demography. Mcclean [8] developed a two stage manpower planning model for personnel behavior. Ugwuowo and McClean [23] reviewed manpower models and the sources of heterogeneity. Vassiliou[24] developed a manpower model for a graded system with Markova process. Srinivasa Rao[14] developed a two graded manpower model with open ended systems. Time-de feyter and marie-anne guerry [20] have reviewed the Markova models in manpower planning involving Markova chains. Esther clara and Srinivasan[7] developed a stochastic model for the expected time to recruitment in a single graded manpower system with two thresholds. Srinivasan and Vasudevan [15] have developed two mathematical models using univariate policy of recruitment based on shock models.

In all the above papers, it is assumed that the recruitment is done in one person at a time. Many researchers developed manpower models for graded system using Markova processes. The graded system is a common phenomenon in several organizations. In many graded manpower systems, the recruitment is done in bulk. That is, many (several) people are recruited at a time. For example, in banks, defense and other government organizations the recruitment is done by a recruitment authority which will recruit several people at a time. In some organizations like defense services, the bulk recruitment is done in both the grades with different recruitment rates in addition to the promotions.

Very little work has been reported in literature regarding graded manpower models with bulk recruitment. In this paper, we develop and analyze a two grade manpower model with direct bulk recruitment in both the grades. Here it is assumed that the organization is having two grades and the bulk recruitment is done in both the grades in addition to the promotions. That is, a random number of employees at a time are recruited for both the grades with different recruitment rates and the recruitment process has been characterized by a compound Poisson processes. The employee once recruited to the first grade and after spending a random period of time he may be promoted to the second grade or leave the organization with certain probability. The employee after spending a random period of time in grade 2, he leaves the organization. This strategy of admitting the bulk recruitment directly to the grade 2 will build internal competition in grade 1 for promotion. Here also it is assumed that the promotion and leaving processes follow Poisson processes. Using the difference differential equations, the joint probability generating function of the number of employees in each grade at a given time‘t’ is derived. The characterizing of the model like, the average number of employees in each grade, the average duration of stay of an employee in each grade, the variance of the number of employees in each grade and the coefficient of variation of an employee in each grade are derived explicitly.

2. Manpower model and transient solution

In this section, consider a two graded manpower model in which the organization is having two grades namely, grade 1 and grade 2. At every recruitment a group of employees are recruited into grade 1 and grade 2. Let us assume that the actual number of employees in grade 1 and grade 2 be ‘X’ and ‘Y’ respectively with probability density functions C_x and C_y . If λ_x is the recruitment rate of batches of sizes x in grade 1 and δ_y is the recruitment rate of batches of sizes y, then the composite recruitment rate λ equals to $\sum_x \lambda_x$ and δ equals to $\sum_y \delta_y$, where the composite recruitment process follows a compound Poisson process. It is also assumed that once an employee is recruited in grade 1, after spending a random duration of time in grade 1, he may be promoted to grade 2 with promotion rate β or may leave the organization with a leaving rate α . It is also assumed that the employee after spending a random duration of time in grade 2, he leaves the organization with a leaving rate γ . It is further assumed that the promotion processes and the leaving processes follow Poisson processes. With these assumptions the schematic diagram representing the manpower system in the organization is shown in figure 1.

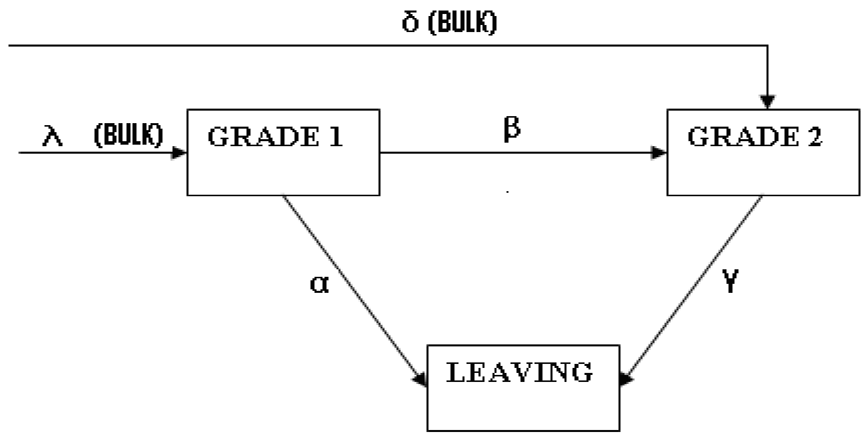

Figure 1: The Schematic diagram of the system.

Let $P_{n,m}(t)$ be the probability that there are n employees in grade 1 and m employees in grade 2 at time‘t’ in the organization.

Then the difference – differential equations governing the model are

$$\frac{\partial}{\partial t}P_{n,m}(t)=\left[-(\lambda+n\alpha+n\beta+m\gamma+\delta)\right]P_{n,m}(t)+(n+1)\alpha P_{n+1,m}(t)+(n+1)\beta P_{n+1,m-1}(t)$$
$$+(m+1)\gamma P_{n,m+1}(t)+\lambda\left[\sum_{i=1}^{n}p_{n-i,m}(t)c_i\right]+\delta\left[\sum_{j=1}^{m}P_{n,m-j}(t)c_j\right],\quad for\ n,m\geq 1 \quad (2.1)$$

$$\frac{\partial}{\partial t}P_{n,0}(t)=\left[-(\lambda+n\alpha+n\beta+\delta)\right]P_{n,0}(t)+(n+1)\alpha P_{n+1,0}(t)+\gamma P_{n,1}(t)+\lambda\left[\sum_{i=1}^{n}p_{n-i,0}(t)c_i\right],$$
$$for\ n\geq 1 \quad (2.2)$$

$$\frac{\partial}{\partial t}P_{0,m}(t)=-(\lambda+m\gamma+\delta)P_{0,m}(t)+\alpha P_{1,m}(t)+\beta P_{1,m-1}(t)+(m+1)\gamma P_{0,m+1}(t)$$
$$+\delta\left[\sum_{j=1}^{m}P_{0,m-j}(t)c_j\right],\quad for\ \ m\geq 1 \quad (2.3)$$

$$\frac{\partial}{\partial t}P_{0,0}(t)=-\left(\lambda+\delta\right)P_{0,0}(t)+\alpha P_{1,0}(t)+\gamma P_{0,1}(t), \quad (2.4)$$

Let $G(Z_1, Z_2; t)$ be the joint probability generating function of $P_{n,m}(t)$, $C(Z_1)$ is the probability generating function of batch size distribution of grade 1 and $C(Z_2)$ is the probability generating function of batch size distribution of grade 2.

Then $G(Z_1, Z_2; t) = \sum_n \sum_m Z_1^n Z_2^m P_{n,m}(t)$, $C(Z_1) = \sum_x C_x Z_1^x$ and $C(Z_2) = \sum_y C_y Z_2^y$ (2.5)

Multiplying the equations (2.1) to (2.4) with corresponding $Z_1^n Z_2^m$ and summing over all n=0, 1, 2,…, m = 0, 1, 2, …, we get

$$\begin{aligned}\frac{\partial}{\partial t}G(Z_1,Z_2;t) &= -\lambda G(Z_1,Z_2;t) - \alpha Z_1 \frac{\partial}{\partial Z_1}G(Z_1,Z_2;t) - \beta Z_1 \frac{\partial}{\partial Z_1}G(Z_1,Z_2;t) \\ &\quad - \gamma Z_2 \frac{\partial}{\partial Z_2}G(Z_1,Z_2;t) - \delta G(Z_1,Z_2;t) + \alpha \frac{\partial}{\partial Z_1}G(Z_1,Z_2;t) + \beta Z_2 \frac{\partial}{\partial Z_1}G(Z_1,Z_2;t) \\ &\quad + \gamma \frac{\partial}{\partial Z_2}G(Z_1,Z_2;t) + \lambda C(Z_1)G(Z_1,Z_2;t) + \delta C(Z_2)G(Z_1,Z_2;t) \qquad (2.6)\end{aligned}$$

After simplification we have

$$\begin{aligned}\frac{\partial}{\partial t}G(Z_1,Z_2;t) = \frac{\partial}{\partial t}G &= \left[\alpha(1-Z_1) + \beta(Z_2 - Z_1)\right]\frac{\partial}{\partial Z_1}G(Z_1,Z_2;t) + \left[\gamma(1-Z_2)\right]\frac{\partial}{\partial Z_2}G(Z_1,Z_2;t) \\ &\quad + \lambda\left[C(Z_1)-1\right]G(Z_1,Z_2;t) + +\delta\left[C(Z_2)-1\right]G(Z_1,Z_2;t) \qquad (2.7)\end{aligned}$$

Solving the equation (2.7) by Lagrangian's method, the auxiliary equations are

$$\frac{dt}{1} = \frac{-dZ_1}{\left[\alpha(1-Z_1)+\beta(Z_2-Z_1)\right]} = \frac{-dZ_2}{\left[\gamma(1-Z_2)\right]} = \frac{dG(Z_1,Z_2;t)}{\left[\lambda(C(Z_1)-1)+\delta(C(Z_2)-1)\right]G(Z_1,Z_2;t)}$$

(2.8)

The initial conditions of the system are

$$P_{N_0,M_0}(0) = 1 \;;\; P_{N_0,M_0}(t) = 0, \quad for\, t > 0 \qquad (2.9)$$

That is initially the organization is having N_0 employees in grade 1 and M_0 employees in grade 2.

Solving the equations (2.8), one can get

$$A=(Z_2-1)e^{-\gamma t},$$
$$B=\left[(Z_1-1)+\frac{\beta}{(\gamma-\alpha-\beta)}(Z_2-1)\right]e^{-(\alpha+\beta)t},$$

$$C=G(Z_1,Z_2;t)\exp\left\{-\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^s A^s B^{r-s}\frac{e^{(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]-\delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y}c_y\binom{y}{u}A^u\frac{e^{\gamma ut}}{\gamma u}\right]\right\} \qquad (2.10)$$

Where, A, B and C are arbitrary constants.

The general solution of the equation (2.10) gives the joint probability generating function of the number of employees in both grades as

$$G(Z_1,Z_2;t)=C\exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^s A^s B^{r-s}\frac{e^{(\gamma s+(\alpha+\beta)(r-s)t}}{(\gamma s+(\alpha+\beta)(r-s)}\right]2+\delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y}C_y\binom{y}{u}A^u\frac{e^{\gamma ut}}{\gamma u}\right]\right\} \qquad (2.11)$$

Using the initial conditions and substituting the value of C in equation (2.11) one can get the joint probability generating function of $p_{n,m}(t)$ as

$$G(Z_1,Z_2;t)=\exp\left\{ \lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^s(Z_2-1)^s \left((Z_1-1)+\frac{\beta}{\gamma-\alpha-\beta}(Z_2-1)\right)^{r-s}\left(\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right)\right] + \delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y}C_y\binom{y}{u}(Z_2-1)^u\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-(1-Z_1)e^{-(\alpha+\beta)t}-\frac{\beta}{\alpha+\beta-\gamma}(1-Z_2)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};$$

$$|Z_1|<1,\ |Z_2|<1 \qquad (2.12)$$

3. Characteristics of the model

The probability that there is no employee in the organization is

$$G_{0,0}(t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta^s(\alpha-\gamma)^{r-s}}{(\gamma-\alpha-\beta)^r}\right)\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}+ \delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y}C_y\binom{y}{u}(-1)^u\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-e^{-(\alpha+\beta)t}-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \qquad (3.1)$$

The probability generating function of the number of employees in grade 1 in the organization is

$$G(Z_1;t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(Z_1-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-(1-Z_1)e^{-(\alpha+\beta)t}\right]^{N_0};|Z_1|<1 \qquad (3.2)$$

The probability that there is no grade 1 employee in the organization is

$$G_{0\bullet}(t)=\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \qquad (3.3)$$

The probability generating function of the number of employees in grade 2 in the organization is

$$G(Z_2;t) = \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^r (Z_2-1)^r \left[\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right] + \delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y \binom{y}{u}(Z_2-1)^u\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-\frac{\beta}{\alpha+\beta-\gamma}(1-Z_2)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};\ |Z_2|<1 \qquad (3.4)$$

The probability that there is no grade 2 employee in the organization is

$$G_{\bullet 0}(t) = \exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r \frac{1-e^{(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right] + \delta\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y\binom{y}{u}(-1)^u\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right\}\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \qquad (3.5)$$

The mean number of employees in grade 1 in the organization is

$$L_1 = \frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty} xC_x\right)\left(1-e^{-(\alpha+\beta)t}\right)+N_0 e^{-(\alpha+\beta)t} \qquad (3.6)$$

The probability that there is at least one employee in grade 1 in the organization is

$$U_1 = 1-G_{0\bullet}(t)$$
$$= 1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x\binom{x}{r}(-1)^r \frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \qquad (3.7)$$

The mean number of employees in grade 2 in the organization is

$$L_2 = \frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right)\left[\left(\frac{1-e^{-rt}}{r}\right)-\left(\frac{e^{-rt}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0e^{-\gamma t}$$
$$+\frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty} y\, \mathrm{C}_y\right)\left(1-e^{-\gamma t}\right), \quad (3.8)$$

The probability that there is at least one grade 2 employee in the organization is

$$U_2 = 1-G_{\bullet 0}(t)$$
$$=1-\exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}C_x\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r\frac{1-e^{(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]\right.$$
$$\left.+\delta\sum_{y=1}^{\infty}\sum_{u=1}^{y}C_y\binom{y}{u}(-1)^u\left(1-\frac{e^{-\gamma ut}}{\gamma u}\right)\right\}\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \quad (3.9)$$

The mean number of employees in the organization is

$$L=\frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty}xC_x\right)\left(1-e^{-(\alpha+\beta)t}\right)+N_0e^{-(\alpha+\beta)t}+\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=1}^{\infty}xC_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]$$
$$+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0e^{-\gamma t}+\frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty}y\,\mathrm{C}_y\right)\left(1-e^{-\gamma t}\right) \quad (3.10)$$

The average duration of stay of an employee in grade 1 is

$$W_1=\frac{L_1}{(\alpha+\beta)(1-G_{0\bullet}(t))}$$

$$=\frac{\frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty}xC_x\right)\left(1-e^{-(\alpha+\beta)t}\right)+N_0e^{-(\alpha+\beta)t}}{(\alpha+\beta)\left\{1-\exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}C_x\binom{x}{r}(-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\right\}} \quad (3.11)$$

The average duration of stay of an employee in grade 2 is

$$W_2 = \frac{L_2}{\gamma(1-G_{\bullet 0}(t))}$$

$$= \frac{\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0 e^{-\gamma t} + \frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty} y\, C_y\right)\left(1-e^{-\gamma t}\right)}{\gamma\left\{\begin{array}{l} 1-\exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r \frac{1-e^{(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]\right. \\ \left. +\delta\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y\binom{y}{u}(-1)^u\left(1-\frac{e^{-\gamma u t}}{\gamma u}\right)\right\}\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \end{array}\right\}} \qquad (3.12)$$

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The variance of the number of employees in grade 1 is

$$V_1 = \frac{\lambda}{2(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)(1-e^{-2(\alpha+\beta)t}) + \frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right)\left(1-e^{-(\alpha+\beta)t}\right)$$

$$+ N_0\left(e^{-(\alpha+\beta)t} - e^{-2(\alpha+\beta)t}\right) \qquad (3.13)$$

The variance of the number of employees in grade 2 is

$$V_2 = \frac{\lambda\beta^2}{(\alpha+\beta-\gamma)^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)+\left(\frac{1-e^{-2rt}}{2r}\right)\right]$$

$$+\delta\left(\sum_{y=1}^{\infty} y(y-1)C_y\left(\frac{1-e^{-2\gamma t}}{2\gamma}\right)\right)+\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]$$

$$+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]$$

$$+M_0\left(e^{-\gamma t}-e^{-(2\gamma)t}\right)+\frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty} y\, C_y\left(1-e^{-\gamma t}\right)\right) \qquad (3.14)$$

The coefficient of variation of the number of employees in grade 1 is

$$\mathrm{CV}_1 = \frac{\sqrt{V_1}}{L_1} \tag{3.15}$$

The coefficient of variation of the number of employees in grade 2 is

$$\mathrm{CV}_2 = \frac{\sqrt{V_2}}{L_2} \tag{3.16}$$

The co-variance between the number of employees in grade 1 and grade 2 is

$$COV\left(N,M\right) = \frac{\lambda\beta}{\alpha+\beta-\gamma}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)-\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)\right]$$

$$+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-2(\alpha+\beta)t}-e^{-(\alpha+\beta+\gamma)t}\right) \tag{3.17}$$

4. Characteristics of the model with uniform batch size distribution

The performance of the model one has to specify the batch size distribution of the bulk recruitment. That is the number of employees recruited at a time is a random variable and follows a specific distribution. Let us assume that the batch size distribution of the bulk recruitment follows uniform distribution with parameters a and b in grade 1 and with parameters c and d in grade 2, respectively.

The probability mass functions of the batch size distribution of grade 1 and grade 2 are

$$C_x = \frac{1}{b-a+1}, \quad x = a, a+1, ..., b, \quad \text{and}$$

$$C_y = \frac{1}{d-c+1}, \quad y = c, c+1, ..., d, \qquad \text{respectively.}$$

The mean number of employees in each batch of grade 1 is $\dfrac{a+b}{2}$

The variance of the batch size of grade 1 is $\dfrac{1}{12}\left[\left(b-a+1\right)^2-1\right]$.

The mean number of employees in each batch of grade 2 is $\dfrac{c+d}{2}$

The variance of the batch size of grade 2 is $\frac{1}{12}\left[\left(d-c+1\right)^2-1\right]$

Substituting the values of C_x and C_y in (12), we get the joint probability generating function of the number of employees in grade 1 and grade 2 is obtained as

$$G(Z_1,Z_2;t)=\exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^{s}(Z_2-1)^{s}\right.\right.$$

$$\left.\left((Z_1-1)+\frac{\beta}{\gamma-\alpha-\beta}(Z_2-1)\right)^{r-s}\left(\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right)\right]$$

$$\left.+\ \delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}(Z_2-1)^{u}\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-(1-Z_1)e^{-(\alpha+\beta)t}-\frac{\beta}{\alpha+\beta-\gamma}(1-Z_2)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0};$$

$$|Z_1|<1\ ,\ |Z_2|<1 \qquad (4.1)$$

The probability that there is no employee in the organization is

$$G_{0,0}(t)=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta^{s}\left(\alpha-\gamma\right)^{r-s}}{(\gamma-\alpha-\beta)^{r}}\right)\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}+\right.$$

$$\left.\delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}(-1)^{u}\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-e^{-(\alpha+\beta)t}-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0} \qquad (4.2)$$

The probability generating function of the number of employees in grade 1 in the organization is

$$G(Z_1;t)=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-(1-Z_1)e^{-(\alpha+\beta)t}\right]^{N_0};|Z_1|<1 \quad (4.3)$$

The probability that there is no grade 1 employee in the organization is

$$G_{0\bullet}(t)=\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0} \quad (4.4)$$

The probability generating function of the number of employees in grade 2 in the organization is

$$G(Z_2;t)=\exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^r(Z_2-1)^r\left[\frac{1-e^{-(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]\right]\right.$$

$$\left.+\delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}(Z_2-1)^u\left(\frac{1-e^{-\gamma ut}}{\gamma u}\right)\right]\right\}$$

$$\left[1-\frac{\beta}{\alpha+\beta-\gamma}(1-Z_2)\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-(1-Z_2)e^{-\gamma t}\right]^{M_0}; \qquad |Z_2|<1$$

(4.5)

The probability that there is no grade 2 employee in the organization is

$$G_{\bullet 0}(t)=\exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r\frac{1-e^{\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]\right.$$

$$\left.+\delta\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}(-1)^u\left(1-\frac{e^{-\gamma ut}}{\gamma u}\right)\right\}\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}$$

(4.6)

The mean number of employees in grade 1 of the organization is

$$L_1=\frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left(1-e^{-(\alpha+\beta)t}\right)+N_0e^{-(\alpha+\beta)t} \quad (4.7)$$

International Journal of Human Resource Management Research
Vol. 1, No. 2, December 2013, PP: 10 -51
Available online at http://acascipub.com/Journals.php

The probability that there is at least one employee in grade 1 in the organization is

$$U_1 = 1 - G_{0\bullet}(t)$$
$$= 1 - \exp\left\{ \lambda \sum_{x=a}^{b} \sum_{r=1}^{x} \left(\frac{1}{b-a+1} \right) \binom{x}{r} (-1)^r \frac{1 - e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r} \right\} \left[1 - e^{-(\alpha+\beta)t} \right]^{N_0} \qquad (4.8)$$

T

he mean number of employees in grade 2 in the organization is

$$L_2 = \frac{\lambda\beta}{\alpha+\beta} \left(\sum_{x=a}^{b} x \left(\frac{1}{b-a+1} \right) \right) \left[\left(\frac{1-e^{-\gamma t}}{\gamma} \right) - \left(\frac{e^{-\gamma t} - e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma} \right) \right] + \frac{N_0 \beta}{\alpha+\beta-\gamma} \left(e^{-\gamma t} - e^{-(\alpha+\beta)t} \right) + M_0 e^{-\gamma t}$$
$$+ \frac{\delta}{\gamma} \left(\sum_{y=c}^{d} y \left(\frac{1}{d-c+1} \right) \right) \left(1 - e^{-\gamma t} \right), \qquad (4.9)$$

The probability that there is at least one grade 2 employee in the organization is

$$U_2 = 1 - G_{\bullet 0}(t)$$
$$= 1 - \exp\left\{ \lambda \left[\sum_{x=a}^{b} \sum_{r=1}^{x} \sum_{s=0}^{r} \left(\frac{1}{b-a+1} \right) \binom{x}{r} \binom{r}{s} (-1)^{3r-s} \left(\frac{\beta}{(\gamma-\alpha-\beta)} \right)^r \frac{1 - e^{(\gamma s + (\alpha+\beta)(r-s))t}}{(\gamma s + (\alpha+\beta)(r-s))} \right] \right.$$
$$\left. + \delta \sum_{y=c}^{d} \sum_{u=1}^{y} \left(\frac{1}{d-c+1} \right) \binom{y}{u} (-1)^u \left(1 - \frac{e^{-\gamma u t}}{\gamma u} \right) \right\} \left[1 - \frac{\beta}{\alpha+\beta-\gamma} \left(e^{-\gamma t} - e^{-(\alpha+\beta)t} \right) \right]^{N_0} \left[1 - e^{-\gamma t} \right]^{M_0} \qquad (4.10)$$

The mean number of employees in the organization is

$$L = \frac{\lambda}{\alpha+\beta} \left(\sum_{x=a}^{b} x \left(\frac{1}{b-a+1} \right) \right) \left(1 - e^{-(\alpha+\beta)t} \right) + N_0 e^{-(\alpha+\beta)t} + \frac{\lambda\beta}{\alpha+\beta} \left(\sum_{x=a}^{b} x \left(\frac{1}{b-a+1} \right) \right)$$
$$\left[\left(\frac{1-e^{-\gamma t}}{r} \right) - \left(\frac{e^{-\gamma t} - e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma} \right) \right] + \frac{N_0 \beta}{\alpha+\beta-\gamma} \left(e^{-\gamma t} - e^{-(\alpha+\beta)t} \right)$$
$$+ M_0 e^{-\gamma t} + \frac{\delta}{\gamma} \left(\sum_{y=c}^{d} y \left(\frac{1}{d-c+1} \right) \right) \left(1 - e^{-\gamma t} \right) \qquad (4.11)$$

The average duration of stay of an employee in grade 1 is

$$W_1 = \frac{L_1}{(\alpha+\beta)(1-G_{0\bullet}(t))}$$

$$= \frac{\frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(1-e^{-(\alpha+\beta)t}\right)+N_0 e^{-(\alpha+\beta)t}}{(\alpha+\beta)\left\{1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(-1)^r\frac{1-e^{-(\alpha+\beta)rt}}{(\alpha+\beta)r}\right\}\left[1-e^{-(\alpha+\beta)t}\right]^{N_0}\right\}} \quad (4.12)$$

The average duration of stay of an employee in grade 2 is

$$W_2 = \frac{L_2}{\gamma(1-G_{\bullet 0}(t))}$$

$$= \frac{\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)+M_0 e^{-\gamma t}+\frac{\delta}{\gamma}\left(\sum_{y=c}^{d} y\left(\frac{1}{d-c+1}\right)\right)\left(1-e^{-\gamma t}\right)}{\gamma\left\{1-\exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r\frac{1-e^{(\gamma s+(\alpha+\beta)(r-s))t}}{(\gamma s+(\alpha+\beta)(r-s))}\right]+\delta\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}(-1)^u\left(1-\frac{e^{-\gamma ut}}{\gamma u}\right)\right\}\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]^{N_0}\left[1-e^{-\gamma t}\right]^{M_0}\right\}}$$

(4.13)

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The variance of the number of employees in grade 1 is

$$V_1 = \frac{\lambda}{2(\alpha+\beta)}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)(1-e^{-2(\alpha+\beta)t})+\frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(1-e^{-(\alpha+\beta)t}\right)$$

$$+N_0\left(e^{-(\alpha+\beta)t}-e^{-2(\alpha+\beta)t}\right) \quad (4.14)$$

The variance of the number of employees in grade 2 is

$$V_2=\frac{\lambda\beta^2}{(\alpha+\beta-\gamma)^2}\left(\sum_{x=a}^{b}x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)-2\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)+\left(\frac{1-e^{-2\gamma t}}{2\gamma}\right)\right]$$

$$+\delta\left(\sum_{y=c}^{d}y(y-1)\left(\frac{1}{d-c+1}\right)\left(\frac{1-e^{-2\gamma t}}{2\gamma}\right)\right)+\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=a}^{b}x\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-\gamma t}}{\gamma}\right)-\left(\frac{e^{-\gamma t}-e^{-(\alpha+\beta)t}}{\alpha+\beta-\gamma}\right)\right]$$

$$+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\left[1-\frac{\beta}{\alpha+\beta-\gamma}\left(e^{-\gamma t}-e^{-(\alpha+\beta)t}\right)\right]$$

$$+M_0\left(e^{-\gamma t}-e^{-(2\gamma)t}\right)+\frac{\delta}{\gamma}\left(\sum_{y=c}^{d}y\left(\frac{1}{d-c+1}\right)\left(1-e^{-\gamma t}\right)\right) \qquad (4.15)$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1=\frac{\sqrt{V_1}}{L_1} \qquad (4.16)$$

The coefficient of variation of the number of employees in grade 2 is

$$CV_2=\frac{\sqrt{V_2}}{L_2} \qquad (4.17)$$

The co-variance between the number of employees in grade 1 and grade 2 is

$$COV(N,M)=\frac{\lambda\beta}{\alpha+\beta-\gamma}\left(\sum_{x=a}^{b}x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1-e^{-(\alpha+\beta+\gamma)t}}{\alpha+\beta+\gamma}\right)-\left(\frac{1-e^{-2(\alpha+\beta)t}}{2(\alpha+\beta)}\right)\right]$$

$$+\frac{N_0\beta}{\alpha+\beta-\gamma}\left(e^{-2(\alpha+\beta)t}-e^{-(\alpha+\beta+\gamma)t}\right) \qquad (4.18)$$

5. Numerical illustration and results

In this section, the behavior of the model is discussed through a numerical illustration. Different values of the parameters are considered for recruitment rate, promotion rates and leaving rates of the system. Since the performance characteristics of the manpower model are highly sensitive with respect to the time, the transient behavior of the model is studied through computing the performance measures with the following set of values for the model parameters.

t = 0.1, 0.5, 0.7, 1, 2;   λ = 1, 3, 5

α = 3, 4, 5; β = 0.9, 1.3, 1.7

δ = 1, 4, 5; γ = 7, 9, 11

a = 1, 2, 3, 6; b = 10, 15, 20, 25

c = 1, 2, 4;   d = 5, 10, 20, 25

N_0 = 500, 700, 900, 1100; M0 = 300, 500, 700, 900

Using the equations (4.7) and (4.9) the average number of employees in the grade 1 and in grade 2 are computed and presented in Table 1. The relationship between the change in parameters and the average number of employees in each grade are shown in figure 2. It is observed that the average number of employees in the grade 1 and grade 2 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.1 to 2 units, the average number of employees in the grade 1 reduces from 551.068 to 5.006, when other parameters are fixed at (2, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (λ, α , β, δ, γ, a, b, c, d, N_0, M_0). Similarly, the average number of employees in grade 2 in the organization reduces from 294.073 to 9.56 for given values of the other parameters. The decrease in the average number of employees in grade 1 is more rapid, when compared to that of grade 2 employees. The number of recruitments per unit time in grade 1 (λ) varies from 1 to 5 units, the average number of employees in grade 1 is increasing from 4.973 to 14.948, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, α, β, δ, γ, a, b, c, d, N_0, M_0). The average number of employees in grade 2 in the organization is increasing from 25.018 to 32.794 for given values of the other parameters. The increase in the average number of employees in grade 1 is moderate, when compared to that of grade 2 employees. As the leaving rate of grade 1 employee (α) varies from 3 to 5 units, the average number of employees in grade 1 reduces from 5.194 to 3.456, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, β, δ, γ, α, b, c, d, N_0, M_0).

Similarly, the average number of employees in grade 2 in the organization reduces from 21.045 to 15.279 for given values of the other parameters. The decrease in average number of employees in grade 1 is moderate, when compared to that of grade 2.

When the promotion rate from grade 1 to grade 2 (β) varies from 0.9 to 1.7 units, the average number of employees in the grade 1 is decreasing from 64.799 to 32.631, when other parameters are fixed at (1, 2, 2, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, δ, γ, a, b, c, d, N_0, M_0). The average number of employees in the grade 2 in the organization is increasing from 28.367 to 32.104 for given values of the other parameters.

The number of recruitments per unit time in grade 2 (δ) varies from 1 to 5 units, the average number of employees in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, γ, a, b, c, d, N_0, M_0). The average number of employees in grade 2 in the organization is increasing from 23.386 to 30.538 for given values of the other parameters.

Table 1: The values of L1 and L2 for different values of parameters

t	**λ**	**α**	**β**	**δ**	**γ**	**a**	**b**	**C**	**d**	**N0**	**M0**	**L1**	**L2**
0.1	2	2	4	3	5	5	25	3	15	1000	100	551.068	294.073
0.5	2	2	4	3	5	5	25	3	15	1000	100	54.538	145.383
0.7	2	2	4	3	5	5	25	3	15	1000	100	19.921	72.639
1	2	2	4	3	5	5	25	3	15	1000	100	7.466	26.962
2	2	2	4	3	5	5	2	3	15	1000	100	5.006	9.56
1	**1**	2	4	3	5	5	25	3	15	1000	100	4.973	25.018
1	**3**	2	4	3	5	5	25	3	15	1000	100	9.96	28.906
1	**5**	2	4	3	5	5	25	3	15	1000	100	14.948	32.794
1	2	**3**	4	3	5	5	25	3	15	1000	100	5.194	21.045
1	2	**4**	4	3	5	5	25	3	15	1000	100	4.084	17.522
1	2	**5**	4	3	5	5	25	3	15	1000	100	3.456	15.279
1	2	2	**0.9**	3	5	5	25	3	15	1000	100	64.799	28.367
1	2	2	**1.3**	3	5	5	25	3	15	1000	100	45.639	31.228
1	2	2	**1.7**	3	5	5	25	3	15	1000	100	32.631	32.104
1	2	2	4	**1**	5	5	25	3	15	1000	100	7.466	23.386
1	2	2	4	**4**	5	5	25	3	15	1000	100	7.466	28.75
1	2	2	4	**5**	5	5	25	3	15	1000	100	7.466	30.538
1	2	2	4	3	**7**	5	25	3	15	1000	100	7.466	13.035
1	2	2	4	3	**9**	5	25	3	15	1000	100	7.466	8.359
1	2	2	4	3	**11**	5	25	3	15	1000	100	7.466	6.234
1	2	2	4	3	5	**1**	25	3	15	1000	100	6.801	26.444
1	2	2	4	3	5	**2**	25	3	15	1000	100	6.968	26.573
1	2	2	4	3	5	**3**	25	3	15	1000	100	7.134	26.703
1	2	2	4	3	5	**6**	25	3	15	1000	100	7.633	27.092
1	2	2	4	3	5	5	**10**	3	15	1000	100	4.973	25.018
1	2	2	4	3	5	5	**15**	3	15	1000	100	5.804	25.666
1	2	2	4	3	5	5	**20**	3	15	1000	100	6.635	26.314
1	2	2	4	3	5	5	**25**	3	15	1000	100	7.466	26.962
1	2	2	4	3	5	5	25	**1**	15	1000	100	7.466	26.366
1	2	2	4	3	5	5	25	**2**	15	1000	100	7.466	26.664
1	2	2	4	3	5	5	25	**4**	15	1000	100	7.466	27.26
1	2	2	4	3	5	5	25	3	**5**	1000	100	7.466	23.982
1	2	2	4	3	5	5	25	3	**10**	1000	100	7.466	25.472
1	2	2	4	3	5	5	25	3	**20**	1000	100	7.466	28.452
1	2	2	4	3	5	5	25	3	**25**	1000	100	7.466	29.942
1	2	2	4	3	5	5	25	3	15	**500**	100	6.227	18.444
1	2	2	4	3	5	5	25	3	15	**700**	100	6.723	21.851
1	2	2	4	3	5	5	25	3	15	**900**	100	7.218	25.258
1	2	2	4	3	5	5	25	3	15	**1100**	100	7.714	28.666
1	2	2	4	3	5	5	25	3	15	1000	**300**	7.466	26.625
1	2	2	4	3	5	5	25	3	15	1000	**500**	7.466	26.962

1	2	2	4	3	5	5	25	3	15	1000	**700**	7.466	31.005
1	2	2	4	3	5	5	25	3	15	1000	**900**	7.466	32.352

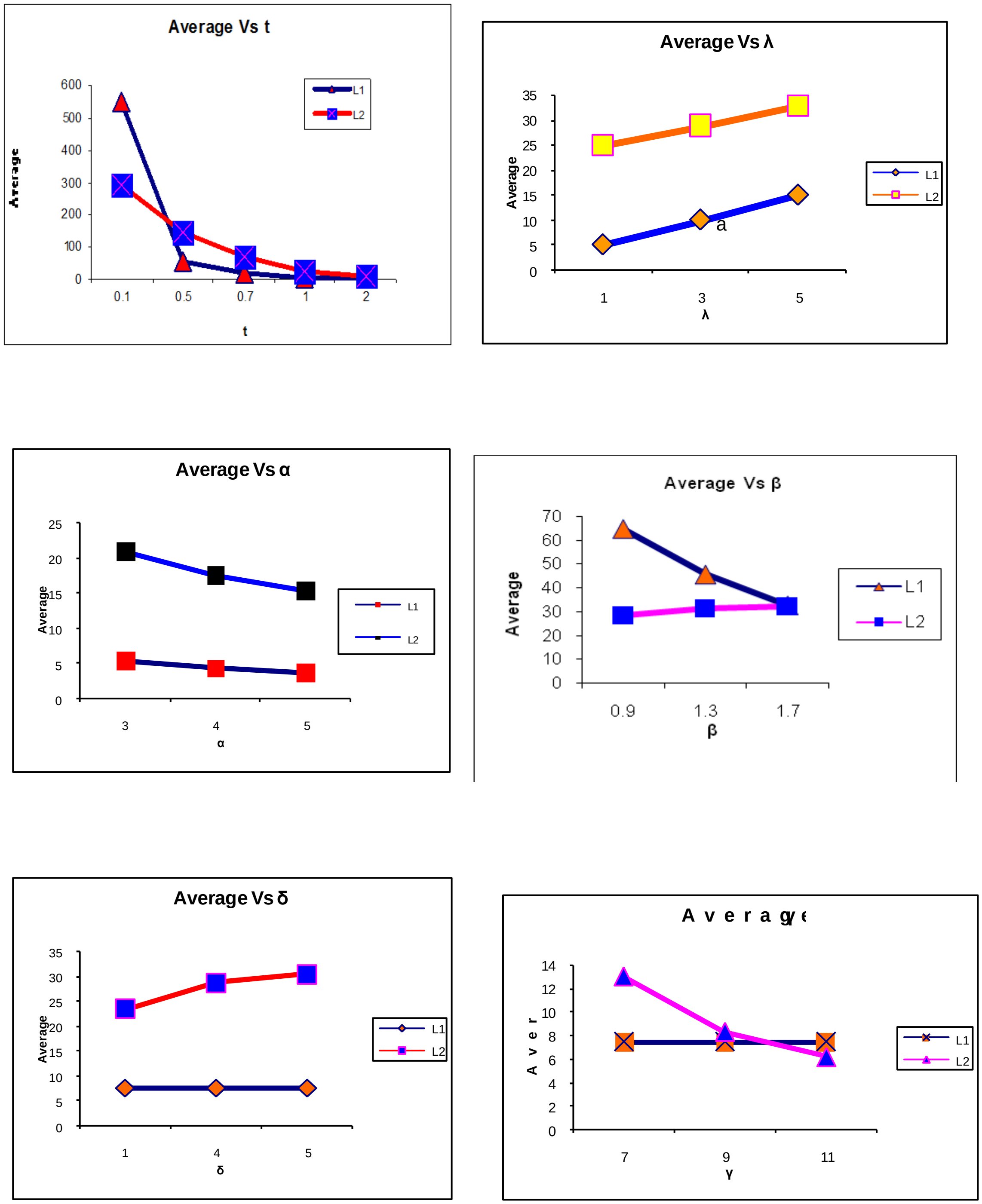

Figure 2: Relationship between Average and t, λ, α, β, δ, γ, a, b, c, d, No and Mo.

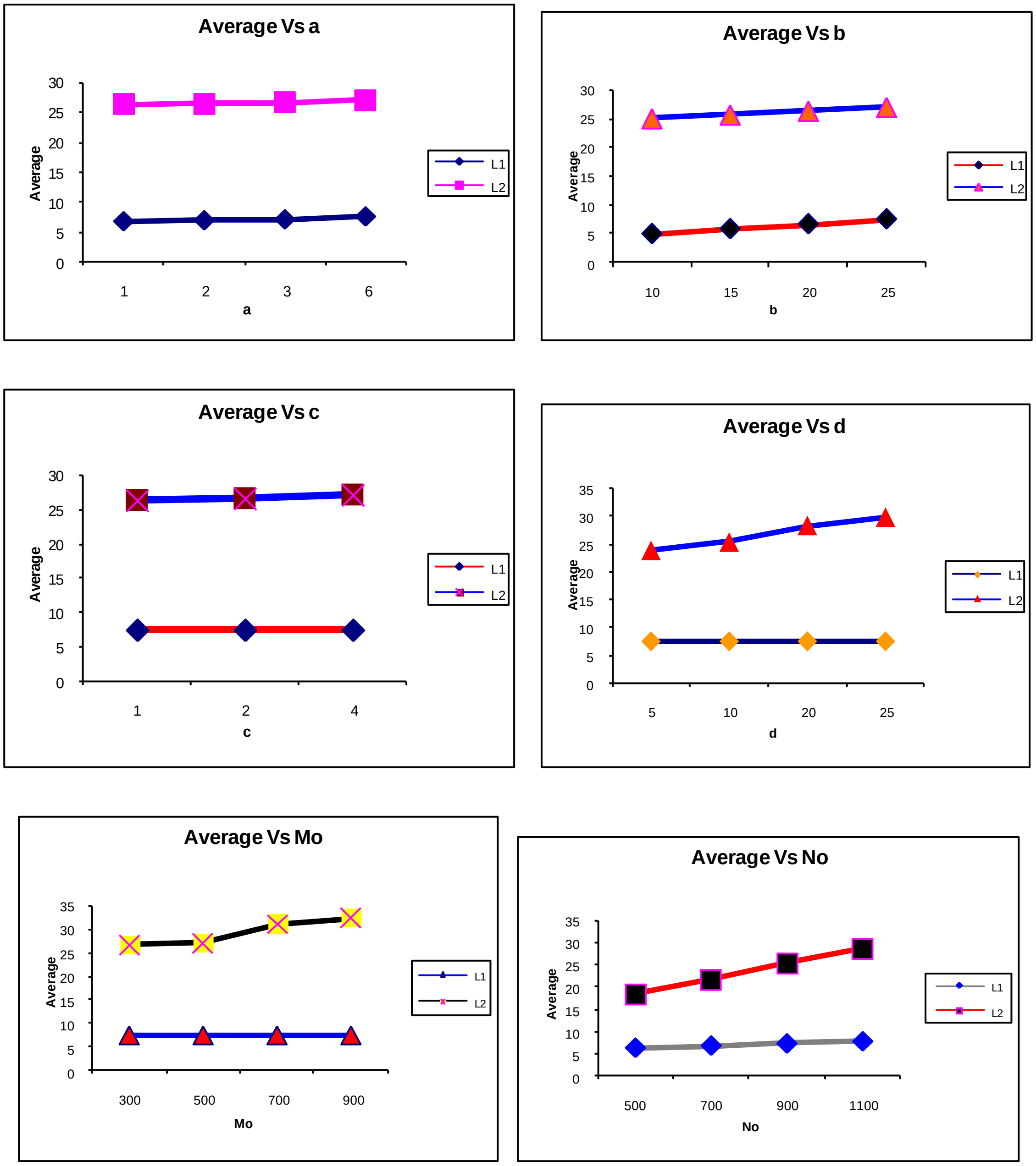

Figure 2: Relationship between Average and t, λ, α, β, δ, γ, a, b, c, d, No and Mo.

As the leaving rate of employee in grade 2 (γ) varies from 7 to 11 units, the average number of employees in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, a, b, c, d, N_0, M_0). The average number of employees in the grade 2 in the organization is decreasing from 13.035 to 6.234 for given values of the other parameters.

As the uniform batch size distribution parameter of grade 1 (a) varies from 1 to 6 units, the average number of employees in the grade 1 is increasing from 6.801 to 7.633, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, γ, b, c, d, $N_{0,}$ Mo). Similarly, the average number of employees in grade 2 in the organization is increasing from 26.334 to 27.092 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 1 (b) varies from 10 to 25 units, the average number of employees in the grade 1 is increasing from 4.973 to 7.466, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 3, 15, 1000, 100), for (t, λ, α, β, δ, γ, a, c, d, $N_{0,}$ Mo). Similarly, the average number of employees in grade 2 in the organization is increasing from 25.018 to 26.962 for given values of the other parameters.

As the batch size distribution parameter of grade 2 (c) varies from 1 to 4 units, the average number of employees in the grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 15, 1000, 100) for (t, λ, α, β, δ, γ, a, b, d, $N_{0,}$ Mo), the average number of employees in grade 2 in the organization is increasing from 26.366 to 27.260 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 2 (d) varies from 5 to 25 units, the average number of employees in grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 1000, 100) for (t, λ, α, β, δ, γ, a, b, c, $N_{0,}$ Mo), the average number of employees in grade 2 in the organization is increasing from 23.982 to 29.942 for given values of the other parameters.

It is further observed that the initial number of employees in grade 1 (N_0) and grade 2 (M_0) in the organization have a vital influence on the average number of employees in grade 1 and the average number of employees in grade 2. As the initial number of employees in grade 1 (No) increases from 500 to 1100, the average number of employees in grade 1 is increases from 6.227 to 7.714, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 100) for (t, λ, α, β, δ γ, a, b, c, d, M_0). Similarly, the average number of employees in grade 2 in the organization is increasing from 18.444 to 28.666 for given values of other parameters. As the initial number of employees in grade 2 (Mo) increases from 300 to 900, the average number of employees in grade 2 in the organization is increasing from 26.625 to 32.352, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 1000) for (t, λ, α, β, δ, γ, a, b, c, d, M_0). As the initial number of employees in grade 2 increases from 300 to 900, the average number of employees in grade 1 remains unchanged for given values of the other parameters.

Using the equations (4.12) and (4.13) the average duration of stay of an employee in the grade 1 and grade 2 in the organization at different values of the parameters are computed and presented in Table 2. The relationship between the parameters and the average duration of stay of an employee in the organization are shown in figure 3.

It is observed that the average duration of stay of an employee in the grade 1 and grade 2 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.3 to 1 units, the average duration of stay of an employee in the grade 1 reduces from 28.245 to 1.281, when other parameters are fixed at (2, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (λ, α , β, δ, γ, α, b, c, d, N_0, M_0). Similarly, the average duration of stay of an employee in grade 2 in the organization reduces from 51.957 to 5.392 for given values of the other parameters. The decrease in the average duration of stay of an employee in grade 1 is more rapid, when compared to that of grade 2 employees.

As the bulk recruitment rate of grade 1 employee (λ) varies from 1 to 5 units, the average duration of stay of an employee in grade 1 is increasing from 0.872 to 2.506, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, α, β, δ, γ, a, b, c, d, N_0, M_0). The average duration of stay of an employee in grade 2 in the organization is increasing from 5.004 to 6.559 for given values of the other parameters. The increase in the average duration of stay of an employee in grade 1 is moderate, when compared to that of grade 2 employees.

As the leaving rate of grade 1 employee (α) varies from 3 to 5 units, the average duration of stay of an employee in grade 1 reduces from 0.884 to 0.676, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, β, δ, γ, a, b, c, d, N_0, M_0). Similarly, the average duration of stay of an employee in grade 2 in the organization reduces from 4.209 to 3.056 for given values of the other parameters. The decrease in average duration of stay of an employee in grade 1 is moderate, when compared to that of grade 2. When the promotion rate from grade 1 to grade 2 (β) varies from 0.9 to 1.7 units, the average duration of stay of an employee in the grade 1 is decreasing from 22.344 to 8.819, when other parameters are fixed at (1, 2, 2, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, δ, γ, a, b, c, d , N_0, M_0), the average duration of stay of an employee in the grade 2 in the organization is increasing from 5.673 to 6.421 for given values of the other parameters.

As the bulk recruitment rate of grade 2 employee (δ) varies from 1 to 5 units, the average duration of stay of an employee in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, γ, a, b, c, d, N_0, M_0). The average duration of stay of an employee in grade 2 in the organization is increasing from 4.677 to 6.108 for given values of the other parameters.

As the leaving rate of employee in grade 2 (γ) varies from 7 to 11 units, the average duration of stay of an employee in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, a, b, c, d, N_0, M_0). The average duration of stay of an employee in grade 2 in the organization is decreasing from 1.862 to 0.583 for given values of the other parameters. As the uniform batch size distribution parameter of grade 1 (a) varies from 1 to 6 units, the average duration of stay of an employee in the grade 1 is increasing from 1.17 to 1.309, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, γ, b, c, d, $N_{0,}$ Mo). Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 5.289 to 5.418 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 1 (b) varies from 10 to 25 units, the average duration of stay of an employee in the grade 1 is increasing from 0.859 to 1.281, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 3, 15, 1000, 100), for (t, λ, α, β, δ, γ, a, c, d, $N_{0,}$ Mo). Similarly the average duration of stay of an employee in grade 2 the organization is increasing from 5.004 to 5.392 for given values of the other parameters. As the batch size distribution parameter of grade 2 (c) varies from 1 to 4 units, the average duration of stay of an employee in the grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 15, 1000, 100) for (t, λ, α, β, δ, γ, a, b, d, $N_{0,}$ Mo). The average duration of stay of an employee in grade 2 in the organization is increasing from 5.273 to 5.452 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 2 (d) varies from 5 to 25 units, the average duration of stay of an employee in grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 1000, 100) for (t, λ, α, β, δ, γ, a, b, c, $N_{0,}$ Mo), the average duration of stay of an employee in grade 2 in the organization is increasing from 4.796 to 5.988 for given values of the other parameters.

It is further observed that the initial number of employees in grade 1 (N_0) and grade 2 (M_0) in the organization have a vital influence on the average duration of stay of an employee in grade 1 and the average duration of stay of an employee in grade 2. As the initial number of employees in grade 1 (No) increases from 500 to 1100, the average

duration of stay of an employee in grade 1 is increasing from 1.153 to 1.313, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 100) for (t, λ, α, β, δ γ, a, b, c, d, M_0).

Similarly, the average duration of stay of an employee in grade 2 in the organization is increasing from 3.689 to 5.733 for given values of other parameters. As the initial number of employees in grade 2 (Mo) increases from 300 to 900, the average duration of stay of an employee in grade 2 in the organization is increasing from 5.325 to 6.47, when

other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 1000) for (t, λ, α, β, δ, γ, a, b, c,

d, M_0), the initial number of employees in grade 2 increases from 300 to 900, the average duration of stay of an employee in grade 1 remains unchanged for given values of the other parameters.

Using the equations (4.14) and (4.15) the variance of the number of employees in the grade 1 and grade 2 in the organization at different time points are computed and presented in Table 3. The relationship between the parameters and the variances of the number of employees in the organization are shown in figure 4.

Table 2: The values of W1 and W2 for different values of parameters

t	λ	α	β	δ	γ	a	b	c	d	N0	M0	W1	W2
0.3	2	2	4	3	5	5	25	3	15	1000	100	28.245	51.957
0.5	2	2	4	3	5	5	25	3	15	1000	100	9.09	29.077
0.9	2	2	4	3	5	5	25	3	15	1000	100	3.32	14.528
1	2	2	4	3	5	5	2	3	15	1000	100	1.281	5.392
1	**1**	2	4	3	5	5	25	3	15	1000	100	0.872	5.004
1	**3**	2	4	3	5	5	25	3	15	1000	100	1.689	5.781
1	**5**	2	4	3	5	5	25	3	15	1000	100	2.506	6.559
1	2	**3**	4	3	5	5	25	3	15	1000	100	0.884	4.209
1	2	**4**	4	3	5	5	25	3	15	1000	100	0.75	3.504
1	2	**5**	4	3	5	5	25	3	15	1000	100	0.676	3.056
1	2	2	**0.9**	3	5	5	25	3	15	1000	100	22.344	5.673
1	2	2	**1.3**	3	5	5	25	3	15	1000	100	13.83	6.246
1	2	2	**1.7**	3	5	5	25	3	15	1000	100	8.819	6.421
1	2	2	4	**1**	5	5	25	3	15	1000	100	1.281	4.677
1	2	2	4	**4**	5	5	25	3	15	1000	100	1.281	5.75
1	2	2	4	**5**	5	5	25	3	15	1000	100	1.281	6.108
1	2	2	4	3	**7**	5	25	3	15	1000	100	1.281	1.862
1	2	2	4	3	**9**	5	25	3	15	1000	100	1.281	0.935
1	2	2	4	3	**11**	5	25	3	15	1000	100	1.281	0.583
1	2	2	4	3	5	**1**	25	3	15	1000	100	1.17	5.289
1	2	2	4	3	5	**2**	25	3	15	1000	100	1.198	5.315
1	2	2	4	3	5	**3**	25	3	15	1000	100	1.226	5.341
1	2	2	4	3	5	**6**	25	3	15	1000	100	1.309	5.418
1	2	2	4	3	5	5	**10**	3	15	1000	100	0.859	5.004
1	2	2	4	3	5	5	**15**	3	15	1000	100	1	5.133
1	2	2	4	3	5	5	**20**	3	15	1000	100	1.14	5.263
1	2	2	4	3	5	5	**25**	3	15	1000	100	1.281	5.392
1	2	2	4	3	5	5	25	**1**	15	1000	100	1.281	5.273
1	2	2	4	3	5	5	25	**2**	15	1000	100	1.281	5.333
1	2	2	4	3	5	5	25	**4**	15	1000	100	1.281	5.452
1	2	2	4	3	5	5	25	3	**5**	1000	100	1.281	4.796
1	2	2	4	3	5	5	25	3	**10**	1000	100	1.281	5.094
1	2	2	4	3	5	5	25	3	**20**	1000	100	1.281	5.69
1	2	2	4	3	5	5	25	3	**25**	1000	100	1.281	5.988
1	2	2	4	3	5	5	25	3	15	**500**	100	1.153	3.689
1	2	2	4	3	5	5	25	3	15	**700**	100	1.193	4.37
1	2	2	4	3	5	5	25	3	15	**900**	100	1.249	5.052
1	2	2	4	3	5	5	25	3	15	**1100**	100	1.313	5.733
1	2	2	4	3	5	5	25	3	15	1000	**300**	1.281	5.325
1	2	2	4	3	5	5	25	3	15	1000	**500**	1.281	5.392
1	2	2	4	3	5	5	25	3	15	1000	**700**	1.281	6.201
1	2	2	4	3	5	5	25	3	15	1000	**900**	1.281	6.47

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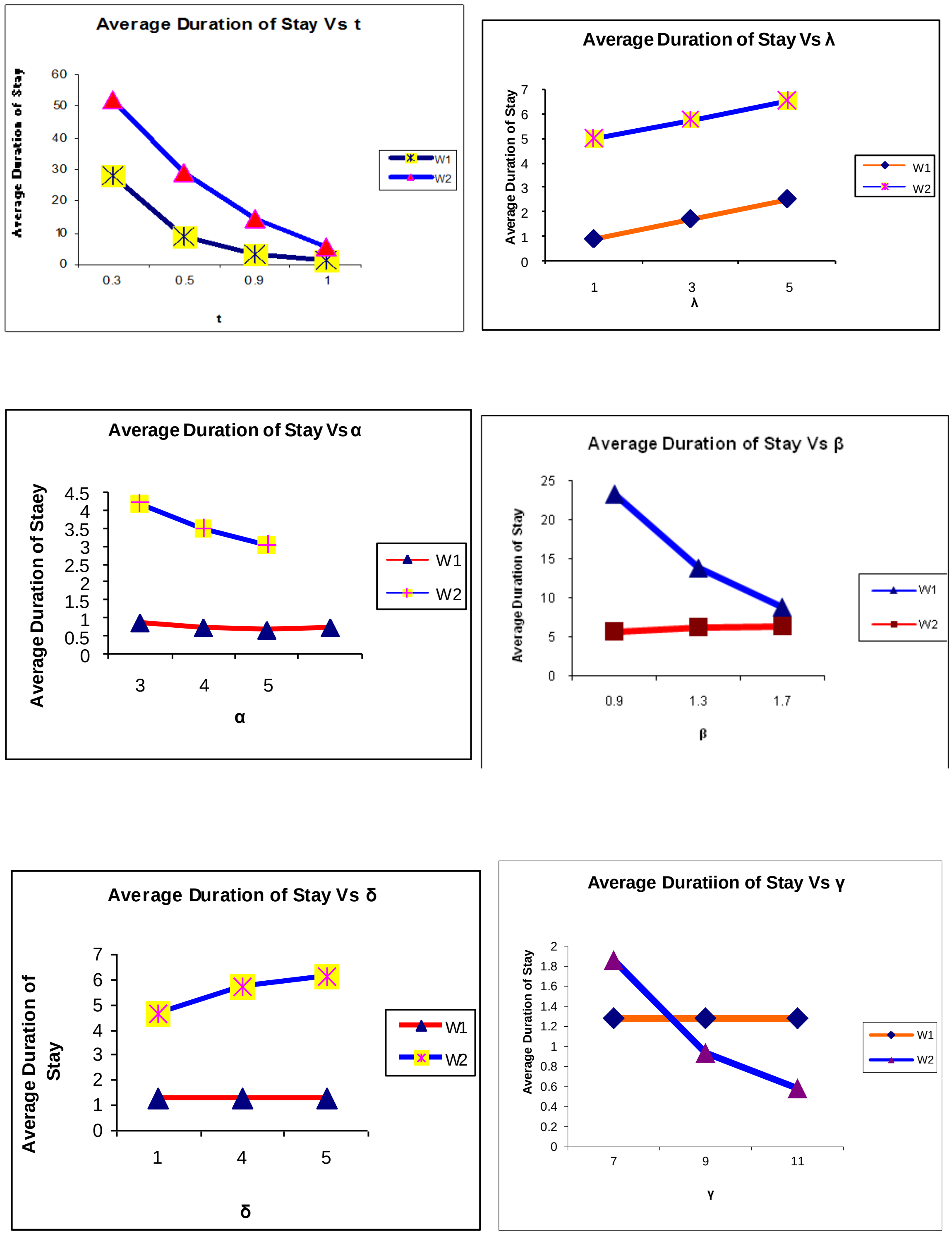


Figure 3: Relationship between Average Duration of Stay and t, λ, α, δ, β, γ, a, b, c, d, No and Mo.

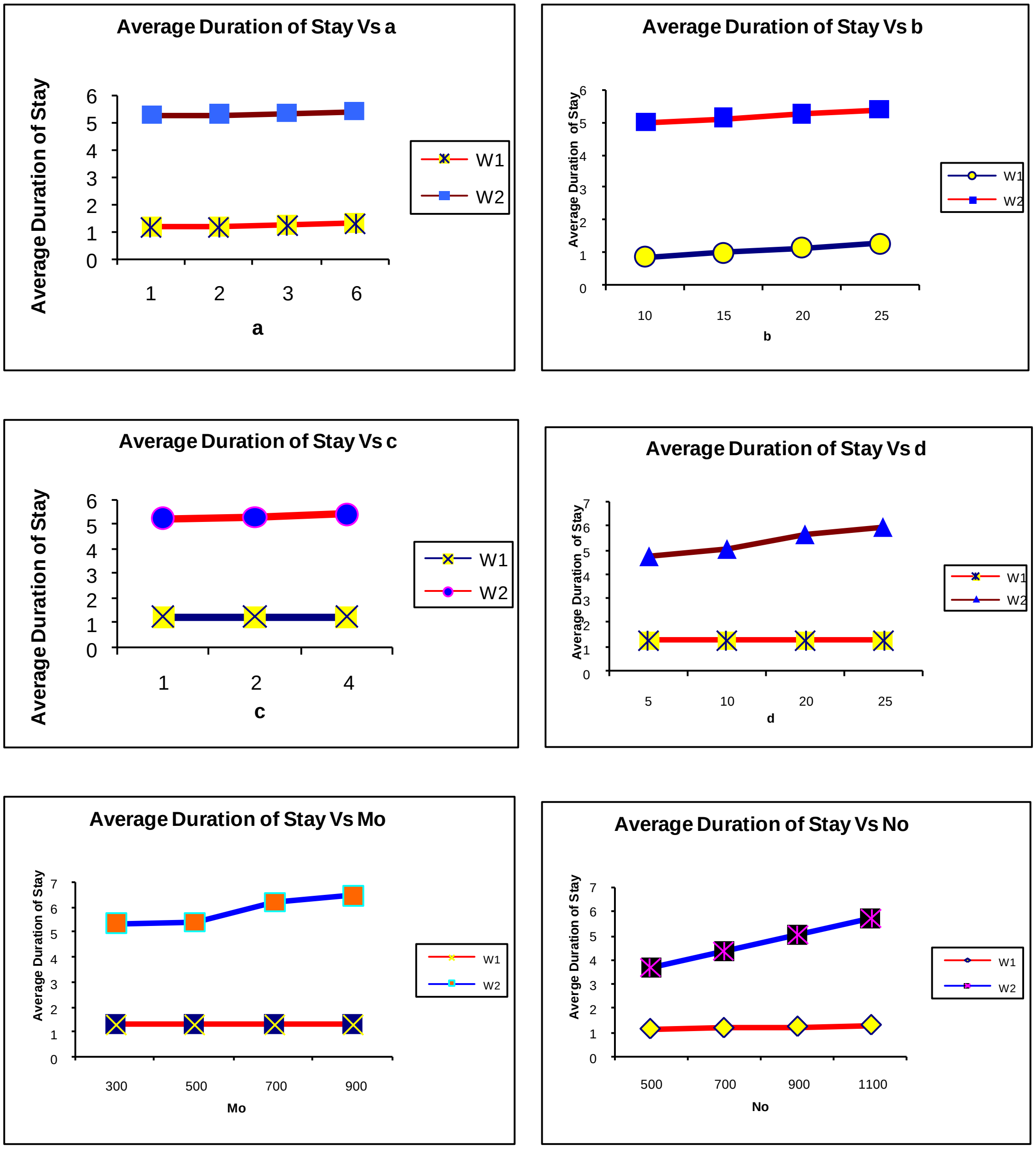


Figure 3: Relationship between Average Duration of Stay and t, λ, α, δ, β, γ, a, b, c, d, No and Mo.

It is observed that the variance of the number of employees in the grade 1 and grade 2 in the organization is highly sensitive with respect to changes in time. As time (t) varies from 0.3 to 2 units, the variance of the number of employees in the grade 1 reduces from 182.136 to 46.117, when other parameters are fixed at (2, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (λ, α , β, δ, γ, a, b, c, d, N_0, M_0). Similarly, the variance of the number of employees in grade 2 in the organization reduces from 865.534 to 693.056 for given values of the other parameters. The decrease in the variance of the number of employees in grade 1 is more rapid, when compared to that of grade 2 employees.

As the bulk recruitment rate of grade 1 employee (λ) varies from 1 to 5 units, the variance of the number of employees in grade 1 is increasing from 25.522 to 117.719, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, α, β, δ, γ, a, b, c, d, N_0, M_0). The variance of the number of employees in grade 2 in the organization is increasing from 379.327 to 1703 for given values of the other parameters. The increase in the variance of the number of employees in grade 1 is moderate, when compared to that of grade 2 employees.

As the leaving rate of grade 1 employee (α) varies from 3 to 5 units, the variance of the number of employees in grade 1 reduces from 40.431 to 30.864, when other parameters are fixed at (1, 2, 4, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, β, δ, γ, α, b, c, d, N_0, M_0). Similarly, the variance of the number of employees in grade 2 in the organization reduces from 187.589 to 68.394 for given values of the other parameters. The decrease in variance of the number of employees in grade 1 is moderate, when compared to that of grade 2. When the promotion rate from grade 1 to grade 2 (β) varies from 0.9 to 1.7 units, the variance of the number of employees in the grade 1 is decreasing from 146.571 to 98.646, when other parameters are fixed at (1, 2, 2, 3, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, δ, γ, a, b, c, d , N_0, M_0), but the variance of the number of employees in the grade 2 in the organization is increasing from 69.155 to 171.15 for given values of the other parameters. As the bulk recruitment rate of grade 2 employee (δ) varies from 1 to 5 units, the variance of the number of employees in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 5, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, γ, a, b, c, d, N_0, M_0). The variance of the number of employees in grade 2 in the organization is increasing from 690.575 to 736.893 for given values of the other parameters. As the leaving rate of employee in grade 2 (γ) varies from 7 to 11 units, the variance of the number of employees in the grade 1 remains unchanged, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, a, b, c, d, N_0, M_0). The variance of the number of employees in the grade 2 in the organization is decreasing from 689.116 to 44.196 for given values of the other parameters.

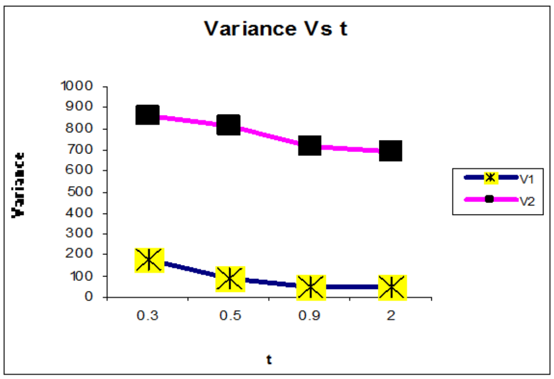
As the uniform batch size distribution parameter of grade 1 (a) varies from 1 to 6 units, the variance of the number of employees in the grade 1 is increasing from 41.462 to 50.626, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 25, 3, 15, 1000, 100) for (t, λ, α, β, δ, γ, b, c, d, $N_{0,}$ Mo). Similarly, the variance of the number of employees in grade 2 in the organization is increasing from 606.529 to 740.51 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 1 (b) varies from 10 to 25 units, the variance of the number of employees in the grade 1 is increasing from 13.577 to 48.571, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 3, 15, 1000, 100), for (t, λ, α, β, δ, γ, a, c, d, $N_{0,}$ Mo). Similarly the variance of the number of employees in grade 2 the organization is increasing from 188.217 to 710.158 for given values of the other parameters.

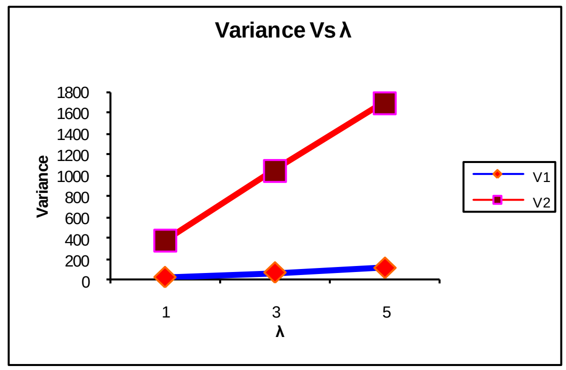
As the batch size distribution parameter of grade 2 (c) varies from 1 to 4 units, the variance of the number of employees in the grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 15, 1000, 100) for (t, λ, α, β, δ, γ, a, b, d, $N_{0,}$ Mo), the variance of the number of employees in grade 2 in the organization is increasing from 706.162 to 712.456 for given values of the other parameters. It is further observed that the batch size distribution parameter of grade 2 (d) varies from 5 to 20 units, the variance of the number of employees in grade 1 remains unchanged, when the other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 1000, 100) for (t, λ, α, β, δ, γ, a, b, c, $N_{0,}$ Mo), the variance of the number of employees in grade 2 in the organization is increasing from 685.179 to 713.734 for given values of the other parameters.

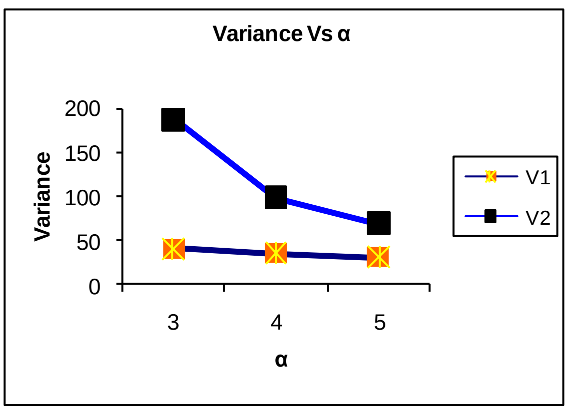
Table 3: The values of V1,V2,CV1,CV2 and COV (N, M) for different values of parameters

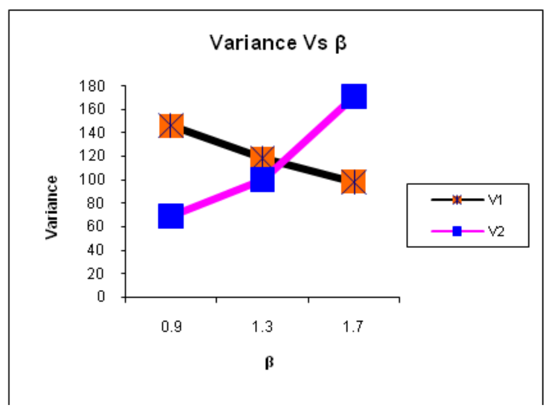
t	λ	α	β	δ	γ	a	b	c	d	N0	M0	V1	V2	CV1	CV2	COV(N,M)
0.3	2	2	4	3	5	5	25	3	15	1000	100	182.136	865.534	0.08	0.113	134.458
0.5	2	2	4	3	5	5	25	3	15	1000	100	93.069	809.71	0.177	0.196	72.146
0.9	2	2	4	3	5	5	25		15	1000	100	50.584	719.416	0.749	0.732	179.183
2	2	2	4	3	5	5	2	3	15	1000	100	46.117	693.056	1.357	2.754	179.311
1	**1**	2	4	3	5	5	25	3	15	1000	100	25.522	379.327	1.016	0.778	89.57
1	**3**	2	4	3	5	5	25	3	15	1000	100	71.62	1046	0.85	1.116	269.961
1	**5**	2	4	3	5	5	25	3	15	1000	100	117.719	1703	0.726	1.258	448.352
1	2	**3**	4	3	5	5	25	3	15	1000	100	40.431	187.589	1.224	0.651	82.14
1	2	**4**	4	3	5	5	25	3	15	1000	100	34.917	98.004	1.447	0.565	50.3533
1	2	**5**	4	3	5	5	25	3	15	1000	100	30.864	68.394	1.607	0.541	35.182
1	2	2	**0.9**	3	5	5	25	3	15	1000	100	146.571	69.155	0.187	0.293	-28.064
1	2	2	**1.3**	3	5	5	25	3	15	1000	100	118.924	100.001	0.239	0.32	-46.443
1	2	2	**1.7**	3	5	5	25	3	15	1000	100	98.646	171.15	0.304	0.407	-74.857
1	2	2	4	**1**	5	5	25	3	15	1000	100	48.571	690.575	0.933	1.124	179.265
1	2	2	4	**4**	5	5	25	3	15	1000	100	48.571	725.313	0.933	0.937	179.265
1	2	2	4	**5**	5	5	25	3	15	1000	100	48.571	736.893	0.933	0.889	179.265
1	2	2	4	3	**7**	5	25	3	15	1000	100	48.571	689.116	0.933	2.014	-151.893
1	2	2	4	3	**9**	5	25	3	15	1000	100	48.571	95.69	0.933	1.17	-43.943
1	2	2	4	3	**11**	5	25	3	15	1000	100	48.571	44.196	0.933	1.066	-23.304
1	2	2	4	3	5	**1**	25	3	15	1000	100	41.462	606.529	0.947	0.931	151.145
1	2	2	4	3	5	**2**	25	3	15	1000	100	43.072	629.77	0.942	0.944	157.448
1	2	2	4	3	5	**3**	25	3	15	1000	100	44.794	654.788	0.938	0.958	164.235
1	2	2	4	3	5	**6**	25	3	15	1000	100	50.626	740.51	0.932	1.004	187.508
1	2	2	4	3	5	5	**10**	3	15	1000	100	13.577	188.217	0.741	0.548	37.45
1	2	2	4	3	5	5	**15**	3	15	1000	100	22.464	317.753	0.817	0.695	72.6
1	2	2	4	3	5	5	**20**	3	15	1000	100	34.129	491.734	0.88	0.843	119.872
1	2	2	4	3	5	5	**25**	3	15	1000	100	48.571	710.158	0.933	0.988	179.265
1	2	2	4	3	5	5	25	**1**	15	1000	100	48.571	706.162	0.933	1.008	179.265
1	2	2	4	3	5	5	25	**2**	15	1000	100	48.571	708.06	0.933	0.998	179.265
1	2	2	4	3	5	5	25	**4**	15	1000	100	48.571	712.456	0.933	0.979	179.265
1	2	2	4	3	5	5	25	3	**5**	1000	100	48.571	685.179	0.933	1.091	179.265
1	2	2	4	3	5	5	25	3	**10**	1000	100	48.571	695.169	0.933	1.035	179.265
1	2	2	4	3	5	5	25	3	**20**	1000	100	48.571	713.734	0.933	0.95	179.265

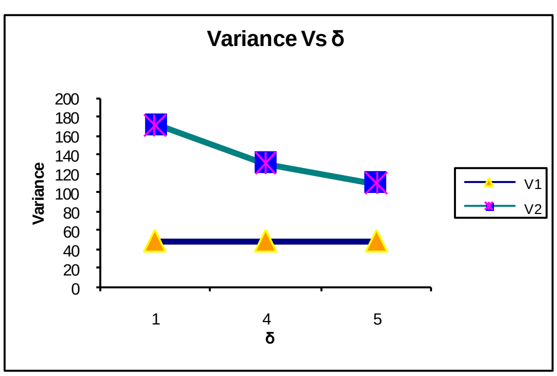
1	2	2	4	3	5	5	25	3	15	**500**	100	47.335	701.785	1.105	1.436	179.286
1	2	2	4	3	5	5	25	3	15	**700**	100	47.829	705.134	1.029	1.215	179.278
1	2	2	4	3	5	5	25	3	15	**900**	100	48.324	708.483	0.963	1.054	179.27
1	2	2	4	3	5	5	25	3	15	**1100**	100	48.818	715.408	0.906	0.933	179.261
1	2	2	4	3	5	5	25	3	15	1000	**300**	48.571	711.497	0.933	0.942	179.265
1	2	2	4	3	5	5	25	3	15	1000	**500**	48.571	712.835	0.933	0.9	179.265
1	2	2	4	3	5	5	25	3	15	1000	**700**	48.571	714.174	0.933	0.862	179.265
1	2	2	4	3	5	5	25	3	15	1000	**900**	48.571	719.088	0.933	0.0829	179.265











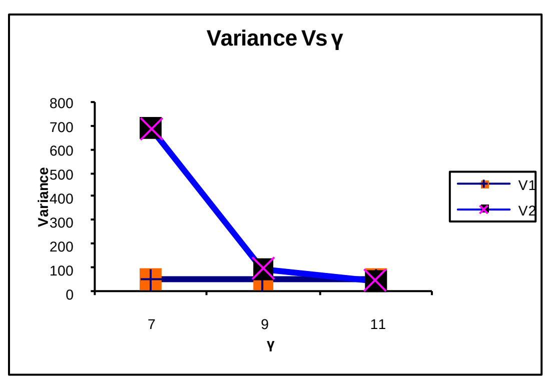


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Figure 4: Relationship between Variance and t, λ, α, β, δ, γ, a, b, c, d, No and Mo.

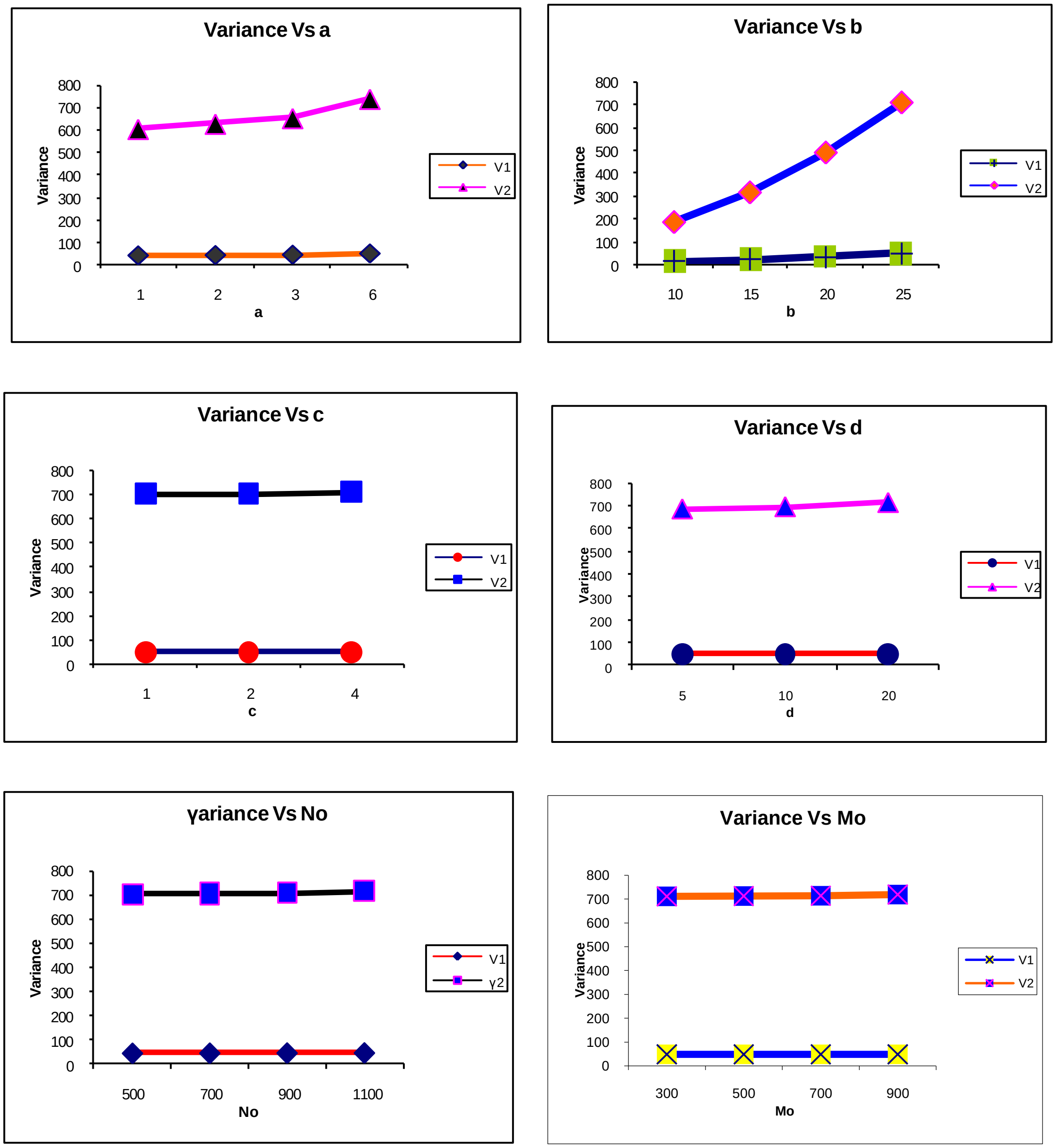

Figure 4: Relationship between Variance and t, λ, α, β, δ, γ, a, b, c, d, No and Mo

It is further observed that the initial number of employees in grade 1 (N_0) and grade 2 (M_0) in the organization have a vital influence on the variance of the number of employees in grade 1 and the variance of the number of employees in grade 2.

As the initial number of employees in grade 1 (No) increases from 500 to 1100, the variance of the number of employees in grade 1 is increasing from 47.335 to 48.818, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 100) for (t, λ, α, β, δ γ, a, b, c, d, M_0). Similarly, the variance of the number of employees in grade 2 in the organization is increasing from 701.785 to 715.408 for given values of other parameters. As the initial number of employees in grade 2 (Mo) increases from 300 to 900, the variance of the number of employees in grade 2 in the organization is increasing from 711.497 to 719.088, when other parameters are fixed at (1, 2, 2, 4, 3, 5, 5, 25, 3, 15, 1000) for (t, λ, α, β, δ, γ, a, b, c, d, M_0). As the initial number of employees in grade 2 increases from 300 to 900, the variance of the number of employees in grade 1 remains unchanged for given values of the other parameters.

6. Steady state analysis of the model

The steady state analysis of the model can be done by assuming that the organization is stable and is under equilibrium condition.

$$\text{i.e. } \lim_{t\to\infty} p_{n,m}(t) = p_{n,m} \text{ and } \lim_{t\to\infty} G(Z_1,Z_2;t) = G(Z_1,Z_2) \qquad (6.1)$$

The joint probability generating functions of the number of grade 1 and grade 2 employees in the organization, when the system is under equilibrium as

$$G(Z_1,Z_2) = \exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^{s}(Z_2-1)^{s}\left((Z_1-1)+\frac{\beta}{\gamma-\alpha-\beta}(Z_2-1)\right)^{r-s}\left(\frac{1}{(\gamma s+(\alpha+\beta)(r-s))}\right)\right]+\delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y\binom{y}{u}\frac{(z_2-1)^u}{\gamma u}\right]\right\};\ for\ |Z_1|<1\ ;\ |Z_2|<1 \qquad (6.2)$$

The probability that there is no employee in the organization is

$$G_{0,0} = \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r} C_x\binom{x}{r}\binom{r}{s}\left(\frac{\beta^s(\alpha-\gamma)^{r-s}}{(\gamma-\alpha-\beta)^r}\right)\frac{1}{(\gamma s+(\alpha+\beta)(r-s))}+\delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y\binom{y}{u}\left(\frac{(-1)^u}{\gamma u}\right)\right]\right\} \qquad (6.3)$$

The probability generating function of the number of employees in grade 1 n the organization is

$$G(Z_1) = \exp\left\{\lambda\sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x\binom{x}{r}(Z_1-1)^r\frac{1}{(\alpha+\beta)r}\right\};|Z_1|<1 \qquad (6.4)$$

The probability that there is no grade 1 employee in the organization is

$$G_{0\bullet} = \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x \binom{x}{r} \frac{(-1)^r}{(\alpha+\beta)r}\right\} \tag{6.5}$$

The probability generating function of the number of employees in grade 2 in the organization is

$$G(Z_2) = \exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s} (-1)^{2r-s} \left(\frac{\beta}{\gamma-\alpha-\beta}\right)^r (Z_2-1)^r \left[\frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right]\right.$$
$$\left. +\delta\left[\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y \binom{y}{u}\frac{(Z_2-1)^u}{\gamma u}\right]\right\}; \qquad |Z_2|<1$$

(6.6)

The probability that there is no grade 2 employee in the organization is

$$G_{\bullet 0} = \exp\left\{\lambda\left[\sum_{x=1}^{\infty}\sum_{r=1}^{x}\sum_{s=0}^{r} C_x \binom{x}{r}\binom{r}{s} (-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r \frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right]\right.$$
$$\left. +\delta\sum_{y=1}^{\infty}\sum_{u=1}^{y} C_y \binom{y}{u}\frac{(-1)^u}{\gamma u}\right\} \tag{6.7}$$

The mean number of employees in grade 1 of the organization is

$$L_1 = \frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right) \tag{6.8}$$

The probability that there is at least one employee in grade 1 in the organization is

$$U_1 = 1 - G_{0\bullet}$$
$$= 1-\exp\left\{\lambda \sum_{x=1}^{\infty}\sum_{r=1}^{x} C_x \binom{x}{r} \frac{(-1)^r}{(\alpha+\beta)r}\right\} \tag{6.9}$$

The mean number of employees in grade 2 in the organization is

$$L_2 = \frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right) + \frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty} y\, C_y\right) \tag{6.10}$$

The probability that there is at least one grade 2 employee in the organization is

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$$U_2 = 1 - G_{\bullet 0}$$

$$= 1 - \exp\left\{ \lambda \left[\sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} C_x \binom{x}{r} \binom{r}{s} (-1)^{3r-s} \left(\frac{\beta}{(\gamma - \alpha - \beta)} \right)^r \frac{1}{(\gamma s + (\alpha + \beta)(r - s))} \right] + \delta \sum_{y=1}^{\infty} \sum_{u=1}^{y} C_y \binom{y}{u} \left(\frac{(-1)^u}{\gamma u} \right) \right\} \qquad (6.11)$$

The mean number of employees in the organization is

$$L = \frac{\lambda}{\alpha + \beta} \left(\sum_{x=1}^{\infty} x\, C_x \right) + \frac{\lambda \beta}{\alpha + \beta} \left(\sum_{x=1}^{\infty} x\, C_x \right) \left(\frac{1}{\gamma} \right) + \frac{\delta}{\gamma} \left(\sum_{y=1}^{\infty} y\, C_y \right) \qquad (6.12)$$

The average duration of stay of an employee in grade 1 is

$$W_1 = \frac{L_1}{(\alpha + \beta)(1 - G_{0\bullet})}$$

$$= \frac{\frac{\lambda}{\alpha + \beta} \left(\sum_{x=1}^{\infty} x\, C_x \right)}{(\alpha + \beta) \left\{ 1 - \exp\left\{ \lambda \sum_{x=1}^{\infty} \sum_{r=1}^{x} C_x \binom{x}{r} \frac{(-1)^r}{(\alpha + \beta) r} \right\} \right\}} \qquad (6.13)$$

The average duration of stay of an employee in grade 2 is

$$W_2 = \frac{L_2}{\gamma (1 - G_{\bullet 0})}$$

$$= \frac{\frac{\lambda \beta}{\alpha + \beta} \left(\sum_{x=1}^{\infty} x\, C_x \right) \left(\frac{1}{\gamma} \right) + \frac{\delta}{\gamma} \left(\sum_{y=1}^{\infty} y\, C_y \right)}{\gamma \left\{ 1 - \exp\left\{ \lambda \left[\sum_{x=1}^{\infty} \sum_{r=1}^{x} \sum_{s=0}^{r} C_x \binom{x}{r} \binom{r}{s} (-1)^{3r-s} \left(\frac{\beta}{(\gamma - \alpha - \beta)} \right)^r \frac{1}{(\gamma s + (\alpha + \beta)(r - s))} \right] + \delta \sum_{y=1}^{\infty} \sum_{u=1}^{y} C_y \binom{y}{u} \frac{(-1)^u}{\gamma u} \right\} \right\}} \qquad (6.14)$$

.

The variance of the number of employees in grade 1 is

$$V_1 = \frac{\lambda}{2(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right) + \frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty} x\, C_x\right) \quad (6.15)$$

The variance of the number of employees in grade 2 is

$$V_2 = \frac{\lambda\beta^2}{(\alpha+\beta-\gamma)^2}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1}{2(\alpha+\beta)}\right) - 2\left(\frac{1}{\alpha+\beta+\gamma}\right) + \left(\frac{1}{2\gamma}\right)\right]$$

$$+\frac{\delta}{2\gamma}\left(\sum_{y=1}^{\infty} y(y-1)C_y\right) + \frac{\lambda\beta}{\gamma(\alpha+\beta)}\left(\sum_{x=1}^{\infty} x\, C_x\right) + \frac{\delta}{\gamma}\left(\sum_{y=1}^{\infty} y\, C_y\right) \quad (6.16)$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \quad (6.17)$$

The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \quad (6.18)$$

The co-variance between the number of employees in grade 1 and grade 2 is

$$COV(N,M) = \frac{\lambda\beta}{\alpha+\beta-\gamma}\left(\sum_{x=1}^{\infty} x(x-1)C_x\right)\left[\left(\frac{1}{\alpha+\beta+\gamma}\right) - \left(\frac{1}{2(\alpha+\beta)}\right)\right] \quad (6.19)$$

7. Steady state analysis of the model with uniform batch size distribution

The number of employees recruit at a time may be more than one and the size is random. The size of a batch in each recruitment follows uniform distribution with parameters a and b in grade 1 and with parameters c and d in grade 2. The probability mass functions of the batch size distribution of grade 1 and grade 2 are

$$C_x = \frac{1}{b-a+1}, \quad x = a, a+1, \ldots., b, \quad \text{and}$$

$$C_y = \frac{1}{d-c+1}, \quad y = c, c+1, ..., d, \qquad \text{respectively.}$$

The mean number of employees in each batch of grade 1 is $\frac{a+b}{2}$

The variance of the batch size of grade 1 is $\frac{1}{12}\left[\left(b-a+1\right)^2-1\right]$.

The mean number of employees in each batch of grade 2 is $\frac{c+d}{2}$

The variance of the batch size of grade 2 is $\frac{1}{12}\left[\left(d-c+1\right)^2-1\right]$

Substituting the values of C_x and C_y in (6.2), we get the joint probability generating function of the number of employees in grade 1 and grade 2 is obtained as

$$G(Z_1, Z_2) = \exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^s (Z_2-1)^s \left((Z_1-1)+\frac{\beta}{\gamma-\alpha-\beta}(Z_2-1)\right)^{r-s}\left(\frac{1}{(\gamma s+(\alpha+\beta)(r-s))}\right)\right] + \delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\frac{(Z_2-1)^u}{\gamma u}\right]\right\}; \; for \; |Z_1|<1 \; ; \; |Z_2|<1 \tag{7.1}$$

The probability that there is no employee in the organization is

$$G_{0,0} = \exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}(-1)^{2r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}\left(\frac{\beta^s\left(\alpha-\gamma\right)^{r-s}}{(\gamma-\alpha-\beta)^r}\right)\frac{1}{(\gamma s+(\alpha+\beta)(r-s)}+ \delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\left(\frac{(-1)^u}{\gamma u}\right)\right]\right\} \tag{7.2}$$

The probability generating function of the number of employees in grade 1 in the organization is

$$G(Z_1) = \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}(Z_1-1)^r \frac{1}{(\alpha+\beta)r}\right\}; |Z_1|<1 \tag{7.3}$$

The probability that there is no grade 1 employee in the organization is

$$G_{0.} = \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}\right\} \tag{7.4}$$

The probability generating function of the number of employees in grade 2 in the organization is

$$G(Z_2) = \exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{2r-s}\left(\frac{\beta}{\gamma-\alpha-\beta}\right)^r (Z_2-1)^r\left[\frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right] + \delta\left[\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\frac{(Z_2-1)^u}{\gamma u}\right]\right\}; \qquad |Z_2|<1 \tag{7.5}$$

The probability that there is no grade 2 employee in the organization is

$$G_{\bullet 0} = \exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^r \frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right] + \delta\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\frac{(-1)^u}{\gamma u}\right\} \tag{7.6}$$

The mean number of employees in grade 1 of the organization is

$$L_1 = \frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right) \tag{7.7}$$

The probability that there is at least one employee in grade 1 in the organization is

$$\begin{aligned} U_1 &= 1 - G_{0\bullet} \\ &= 1-\exp\left\{\lambda \sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^r}{(\alpha+\beta)r}\right\} \end{aligned} \tag{7.8}$$

The mean number of employees in grade 2 in the organization is

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$$L_2 = \frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=a}^{b} x \left(\frac{1}{b-a+1}\right)\right) + \frac{\delta}{\gamma}\left(\sum_{y=c}^{d} y \left(\frac{1}{d-c+1}\right)\right) \tag{7.9}$$

The probability that there is at least one grade 2 employee in the organization is

$$\begin{aligned} U_2 &= 1 - G_{\bullet 0} \\ &= 1 - \exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^{r}\frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right]\right. \\ &\qquad \left. + \delta\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\left(\frac{(-1)^{u}}{\gamma u}\right)\right\} \end{aligned} \tag{7.10}$$

The mean number of employees in the organization is

$$L = \frac{\lambda}{\alpha+\beta}\left(\sum_{x=1}^{\infty} xC_x\right) + \frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(\frac{1}{\gamma}\right) + \frac{\delta}{\gamma}\left(\sum_{y=c}^{d} y \left(\frac{1}{d-c+1}\right)\right) \tag{7.11}$$

The average duration of stay of an employee in grade 1 is

$$\begin{aligned} W_1 &= \frac{L_1}{(\alpha+\beta)(1-G_{0\bullet})} \\ &= \frac{\frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)}{(\alpha+\beta)\left\{1-\exp\left\{\lambda\sum_{x=a}^{b}\sum_{r=1}^{x}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\frac{(-1)^{r}}{(\alpha+\beta)r}\right\}\right\}} \end{aligned} \tag{7.12}$$

The average duration of stay of an employee in grade 2 is

$$W_2 = \frac{L_2}{\gamma(1-G_{\bullet 0})}$$

$$= \frac{\frac{\lambda\beta}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)\left(\frac{1}{\gamma}\right)+\frac{\delta}{\gamma}\left(\sum_{y=c}^{d} y\left(\frac{1}{d-c+1}\right)\right)}{\gamma\left\{\begin{array}{l}1-\exp\left\{\lambda\left[\sum_{x=a}^{b}\sum_{r=1}^{x}\sum_{s=0}^{r}\left(\frac{1}{b-a+1}\right)\binom{x}{r}\binom{r}{s}(-1)^{3r-s}\left(\frac{\beta}{(\gamma-\alpha-\beta)}\right)^{r}\frac{1}{(\gamma s+(\alpha+\beta)(r-s)}\right]\right.\\ \left.+\delta\sum_{y=c}^{d}\sum_{u=1}^{y}\left(\frac{1}{d-c+1}\right)\binom{y}{u}\frac{(-1)^{u}}{\gamma u}\right\}\end{array}\right\}} \qquad (7.13)$$

.

The variance of the number of employees in grade 1 is

$$V_1 = \frac{\lambda}{2(\alpha+\beta)}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)+\frac{\lambda}{\alpha+\beta}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right) \qquad (7.14)$$

The variance of the number of employees in grade 2 is

$$V_2 = \frac{\lambda\beta^2}{(\alpha+\beta-\gamma)^2}\left(\sum_{x=a}^{b} x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1}{2(\alpha+\beta)}\right)-2\left(\frac{1}{\alpha+\beta+\gamma}\right)+\left(\frac{1}{2\gamma}\right)\right]$$

$$+\frac{\delta}{2\gamma}\left(\sum_{y=c}^{d} y(y-1)\left(\frac{1}{d-c+1}\right)\right)+\frac{\lambda\beta}{\gamma(\alpha+\beta)}\left(\sum_{x=a}^{b} x\left(\frac{1}{b-a+1}\right)\right)+\frac{\delta}{\gamma}\left(\sum_{y=c}^{d} y\left(\frac{1}{d-c+1}\right)\right) \qquad (7.15)$$

The coefficient of variation of the number of employees in grade 1 is

$$CV_1 = \frac{\sqrt{V_1}}{L_1} \qquad (7.16)$$

The coefficient of variation of the number of employees in grade 2 is

$$CV_2 = \frac{\sqrt{V_2}}{L_2} \qquad (7.17)$$

The co-variance between the number of employees in grade 1 and grade 2 is

$$COV\left(N,M\right)=\frac{\lambda\beta}{\alpha+\beta-\gamma}\left(\sum_{x=a}^{b}x(x-1)\left(\frac{1}{b-a+1}\right)\right)\left[\left(\frac{1}{\alpha+\beta+\gamma}\right)-\left(\frac{1}{2(\alpha+\beta)}\right)\right] \tag{7.18}$$

Using the equations (7.7), (7.9), (7.12), (7.13), (7.14), (7.15), (7.16), and (7.17), the average number of employees in grade 1 , the average number of employees in grade 2, the average duration of stay of an employees in grade 1, the average duration of stay of an employees in grade 2 , the variance of the number of employees in grade 1, the variance of the number of employees in grade 2 and the coefficient of variation of the number of employees in grade 1 grade 2 are computed and

presented in Table 4, for different values of the parameters as λ= 1, 3, 5; α = 3, 4, 5; β = 6, 7, 8; δ = 1, 4, 5; γ= 7, 9, 11; a= 1, 2, 3, 6; b= 10, 15, 20; c= 1, 2, 4; d= 5, 10, 20; No= 500, 700, 900, 1100 and Mo=300, 500, 700, 900.

Table 4: Values of L1, L2, W1, W2, V1, V2, CV1, CV2 and COV (N, M) for different values of the parameters

λ	α	β	γ	δ	a	b	c	d	N0	M0	L1	L2	W1	W2	V1	V2	CV1	CV2	COV(N,M)
1	2	4	5	3	5	25	3	15	1000	100	2.50	7.400	1.001	1.48	23.056	362.007	1.921	2.571	89.614
3	2	4	5	3	5	25	3	15	1000	100	7.50	11.40	1.561	2.28	69.169	1024	1.109	2.807	269.008
5	2	4	5	3	5	25	3	15	1000	100	12.5	15.40	2.235	3.08	115.278	1686	0.859	2.666	448.402
2	**3**	4	5	3	5	25	3	15	1000	100	4.286	12.429	1.016	2.58	38.524	175.514	1.467	1.501	82.151
2	**4**	4	5	3	5	25	3	15	1000	100	3.75	8.40	0.846	1.777	34.583	88.961	1.568	1.123	50.536
2	**5**	4	5	3	5	25	3	15	1000	100	3.333	8.067	0.723	1.716	30.741	61.231	1.663	0.97	35.183
2	2	**0.9**	5	3	5	25	3	15	1000	100	10.345	7.262	3.999	1.538	95.402	48.532	0.944	0.959	-26.936
2	2	**1.3**	5	3	5	25	3	15	1000	100	9.091	7.764	3.208	1.631	83.838	77.133	1.007	1.131	-45.604
2	2	**1.7**	5	3	5	25	3	15	1000	100	8.108	8.157	2.655	1.707	74.775	147.831	1.008	1.491	-74.288
2	2	4	**7**	3	5	25	3	15	1000	100	5.00	6.714	1.264	0.959	46.111	682.838	1.358	3.892	-151.878
2	2	4	**9**	3	5	25	3	15	1000	100	5.00	5.222	1.264	0.679	46.111	92.564	1.358	1.842	-43.935
2	2	4	**11**	3	5	25	3	15	1000	100	5.00	4.273	1.264	0.482	46.111	42.239	1.358	1.521	-23.299
2	2	4	5	**1**	5	25	3	15	1000	100	5.00	5.80	1.264	1.16	46.111	672.096	1.358	4.47	179.311
2	2	4	5	**4**	5	25	3	15	1000	100	5.00	11.20	1.264	2.24	46.111	703.296	1.358	2.368	179.311
2	2	4	5	**5**	5	25	3	15	1000	100	5.00	13.00	1.264	2.6	46.111	713.696	1.358	2.055	179.311
2	2	4	5	3	**1**	25	3	15	1000	100	4.333	8.867	1.15	1.773	39.00	589.525	1.441	2.738	151.189
2	2	4	5	3	**2**	25	3	15	1000	100	4.50	9.00	1.175	1.8	40.611	612.496	1.416	2.75	157.492
2	2	4	5	3	**3**	25	3	15	1000	100	4.667	9.133	1.203	1.827	42.333	637.518	1.394	2.765	164.28
2	2	4	5	3	**6**	25	3	15	1000	100	5.167	9.533	1.296	1.907	48.167	723.252	1.343	2.821	187.553
2	2	4	5	3	5	**10**	3	15	1000	100	2.50	7.40	0.713	1.575	11.111	170.896	1.333	1.767	37.492
2	2	4	5	3	5	**15**	3	15	1000	100	3.333	8.067	0.9	1.702	20.00	300.452	1.342	2.149	72.644
2	2	4	5	3	5	**20**	3	15	1000	100	4.167	8.733	1.084	1.814	31.667	474.452	1.351	2.494	119.917
2	2	4	5	3	5	25	**1**	15	1000	100	5.00	8.80	1.264	1.76	46.111	688.896	1.358	2.983	179.311

2	2	4	5	3	5	25	**2**	15	1000	100	5.00	9.10	1.264	1.82	46.111	690.796	1.358	2.888	179.311
2	2	4	5	3	5	25	**4**	15	1000	100	5.00	9.70	1.264	1.94	46.111	695.196	1.358	2.718	179.311
2	2	4	5	3	5	25	3	**5**	1000	100	5.00	6.40	1.264	1.28	46.111	667.896	1.358	4.038	179.311
2	2	4	5	3	5	25	3	**10**	1000	100	5.00	7.90	1.264	1.58	46.111	677.896	1.358	3.296	179.311
2	2	4	5	3	5	25	3	**20**	1000	100	5.00	10.90	1.264	2.18	46.111	712.896	1.358	2.45	179.311
2	2	4	5	3	5	25	3	15	**500**	100	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	**700**	100	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	**900**	100	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	1000	100	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	1000	**300**	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	1000	**500**	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	1000	**700**	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311
2	2	4	5	3	5	25	3	15	1000	**900**	5.00	9.40	1.264	1.88	46.111	692.896	1.358	2.8	179.311

From Table 4, it is observed that as the recruitment rate from grade 1 (λ) varies from 1 to 5, the values of L_1, L_2, W_1, W_2, V_1 and V_2 are increasing whereas the values of CV_1 and CV_2 are decreasing for fixed values of the other parameters. As the leaving rate from grade 1 (α) varies from 3 to 5, the values of L_1, L_2, W_1, W_2, V_1, V_2 and CV_2 are decreasing where as the value of CV_1 is increasing for fixed values of other parameters. As the promotion rate (β) from grade 1 to grade 2 varies from 0.9 to 1.7, the values of L_1, W_1 and V_1 are decreasing where as the values of L_2, W_2, V_2, CV_1 and CV_2 are increasing for fixed values of the other parameters. As recruitment rate from grade 2 (δ) varies from 1 to 5, the values of L_2, W_2, V_2 and CV_2 are increasing for fixed values of other parameters, but there is no influence on L_1, W_1, V_1 and CV_1.As the leaving rate from grade 2 (γ) varies from 7 to 11, the values of L_2, W_2, V_2 and CV_2 are decreasing for fixed values of other parameters, but there is no influence on L_1, W_1, V_1 and CV_1.

As the batch size distribution parameter (a) varies from 1 to 6, the values of L_1, L_2, W_1, W_2, V_1, V_2 and CV_2 are increasing where as the values of CV_1 is decreasing for fixed values of other parameters. As the batch size distribution parameter (b) varies from 10 to 20, the values of L_1 L_2, W_1, W_2, V_1, V_2, CV_1 and CV_2 are increasing for fixed values of other parameters. As the batch size distribution parameter of grade 2 (c) varies from 1 to 4, the values of L_2, W_2, V_2, and CV_2 are increasing for fixed values of other parameters, but there is no influence on L_1, W_1, V_1, , CV_1 and COV(N, M).

As the batch size distribution parameter of grade 2 (d) varies from 5 to 20, the values of L_2, W_2, V_2, and CV_2 are increasing for fixed values of other parameters, but there is no influence on L_1, W_1, V_1, CV_1 and COV(N, M). From this analysis it is observed that the two grade manpower system has a significant influence on all performance measures of the model. It is further observed that the performance measures are highly sensitive towards smaller values of time. Hence, it is optimal to consider the graded manpower model and evaluate the performance under transient conditions.

8. Conclusion

This paper deals with the development and analysis of two graded manpower model with bulk recruitment to both grades. In this paper, it is assumed that the recruitment process of batches follows a Poisson process and hence the recruitment of an employee into the organization is characterized with a compound Poisson process. The batch size distribution is follows uniform distribution. A numerical study reveals that this graded manpower model is capable of predicting the performance measures more close to the reality. It is observed that time t has significant influence on all performance measures of manpower model. It is further observed that the bulk recruitment and promotion rates have significant influence on the system behavior. It is observed that there is a variation between transient and steady state behavior of the model. It is interesting to note that this two grade manpower model includes some of the earlier models as particular cases for specific or limiting values of the parameters. It is also possible to develop graded manpower model with bulk recruitment having non-Morkovian promotion and leaving processes which require further investigations.

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